***Parallel Cooperative Optimization* Research Group**

Metaheuristics for Multi-Objective Optimization

E-G. Talbi, S. Cahon

Laboratoire d'Informatique Fondamentale de Lille

CNRS

INRIA

Outline

- Multi-Objective Optimization: Definitions, Problems, etc
- A taxonomy of the resolution methods
- Landscapes and performance analysis
- Conclusions

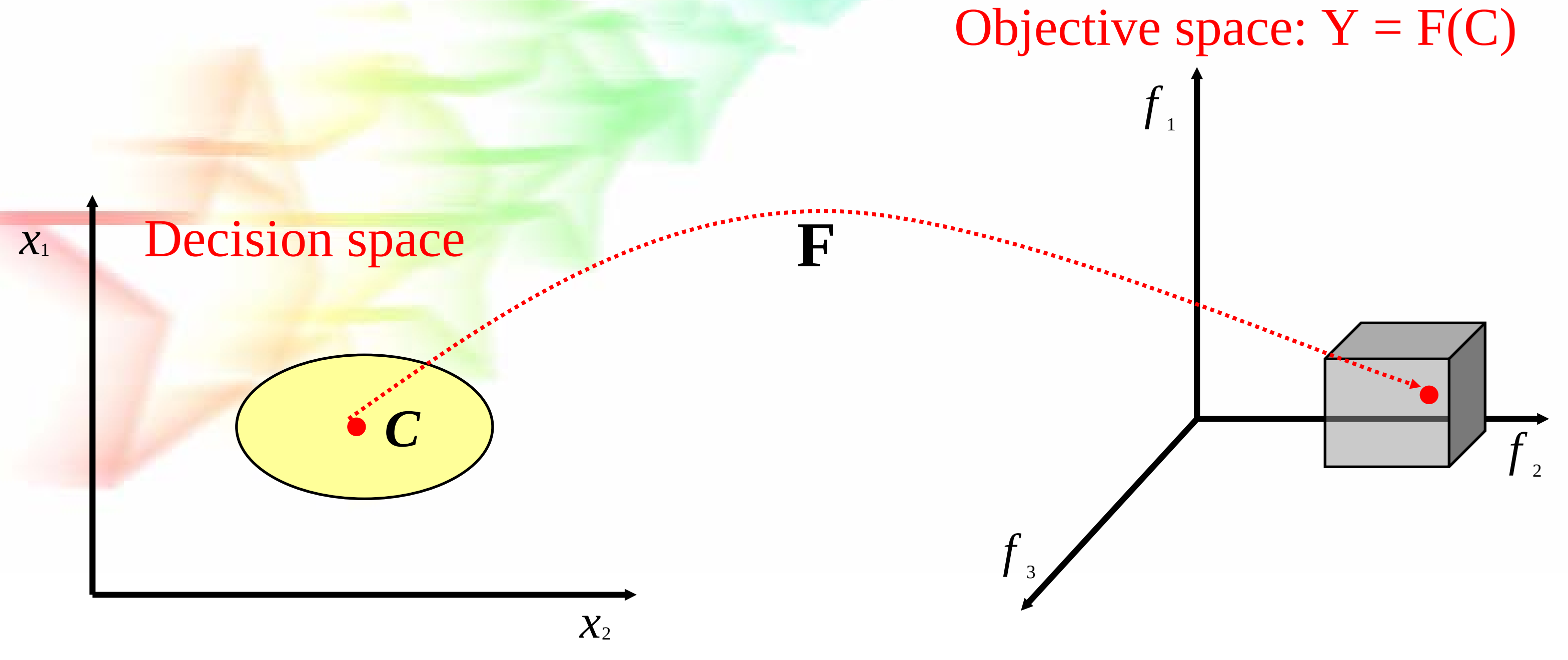
Multi-Objective Optimization

- Many areas of industry are concerned (Telecommunication, Transport, Aeronautics, etc)
- Roots in the 19th century in economy from Edgeworth and Pareto (Management, Engineering, etc)
- Linear/Non linear multi-objective Optimization [Steuer 86, White 90].
- Multi-objective Combinatorial Optimization

Multi-objective optimization

(PMO) $\begin{cases} \text{Min } F(x) = (f_1(x), f_2(x), \ldots, f_n(x)) & n \geq 2 \\ \text{st } x \in C \end{cases}$

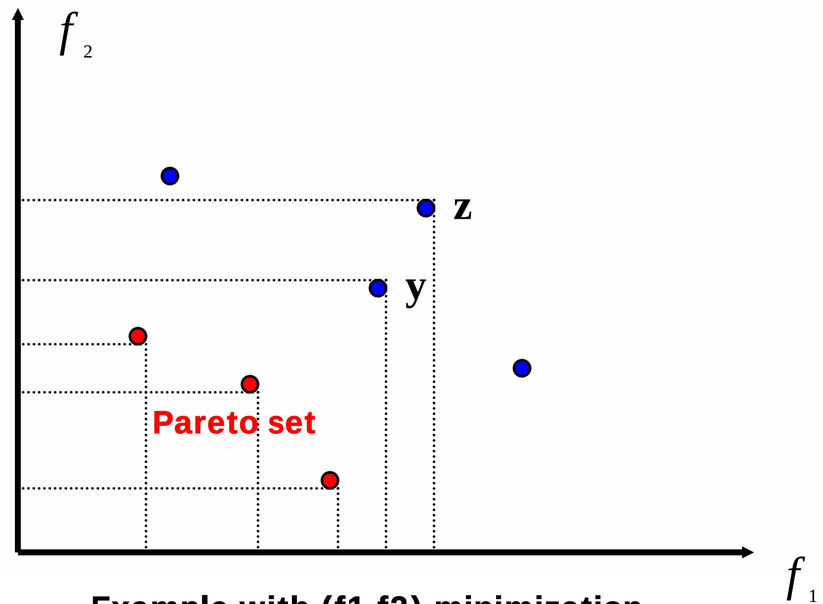
Decision variables $x = (x_1, x_2, \ldots, x_k)$



Multi-objective optimization

- Dominance
  - y dominates z if and only if $\forall i \in [1, ..., n], y_i \leq z_i$
  - and $\exists i \in [1, ..., n], y_i < z_i$
- Non-dominated solution
  - A solution x is non dominated if a solution which dominates x does not exist

- Non-dominated solution (eligible, efficient, non inferior, Pareto optimal)
- Dominated feasible solution

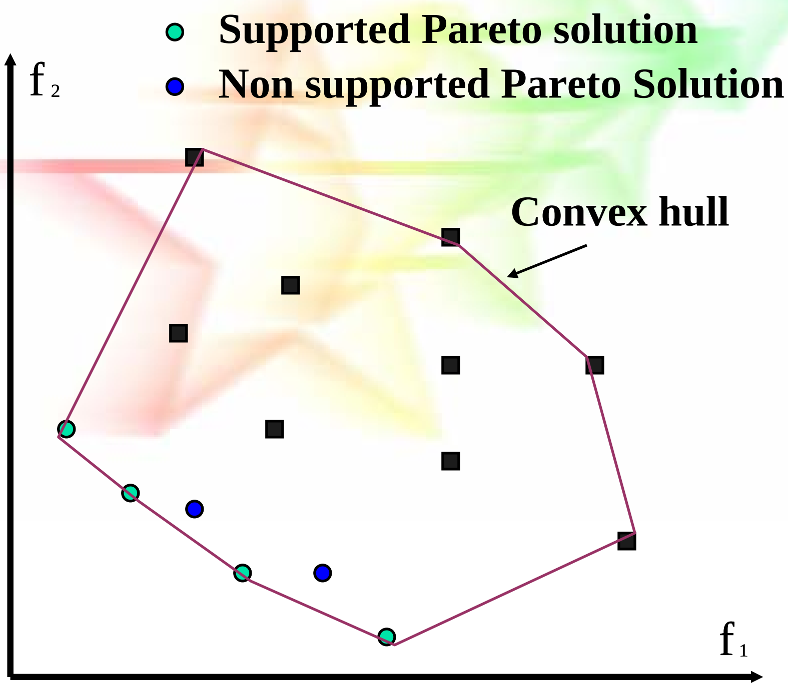


Example with (f1,f2) minimization

➔Goal: Find a good quality and well diversified set of Pareto solutions

Definitions

- Supported/non supported solutions
- Optimal vector y* and the reference vector z* $\quad y^*/y_i^* = \min\left(f_i(x)\right)$

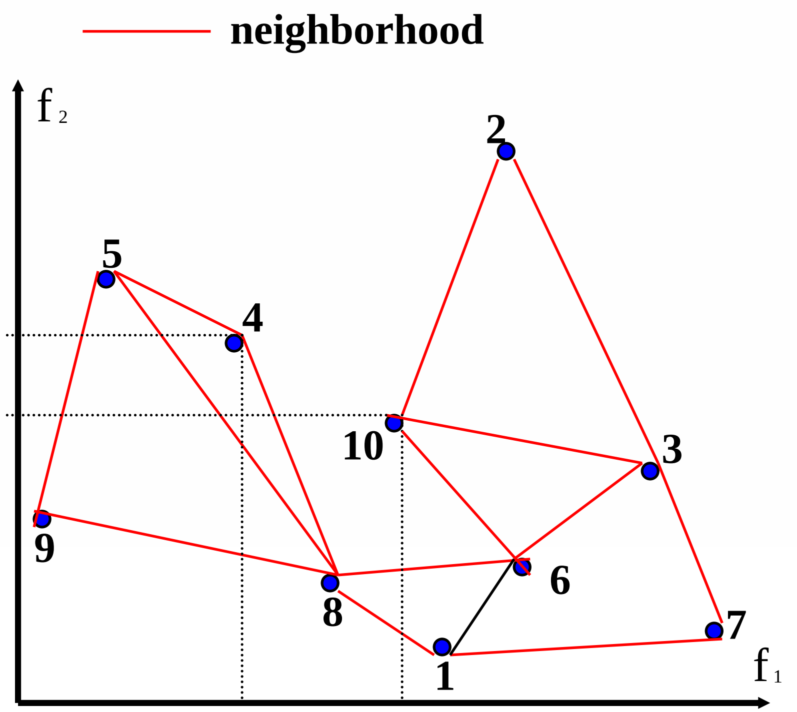


- Locally Pareto optimal

Neighborhood N : Local search

$$N : C \to P(C)$$

1, 8, 9 : Pareto optimal
4, 10 : Locally Pareto optimal



Main issues

- What is optimality ? Partial order, dynamic environment, the last choice belongs to the decision maker
- Number of Pareto solutions increases function of the dimension(s) of the tackled problems and the number of objectives
- For non convex MOPs, Pareto solutions are located on the frontiers and in the convex hulls

Cooperation solver – decision maker

- **A priori :** **Preference** ⟶ **Search** **(Usefulness)**

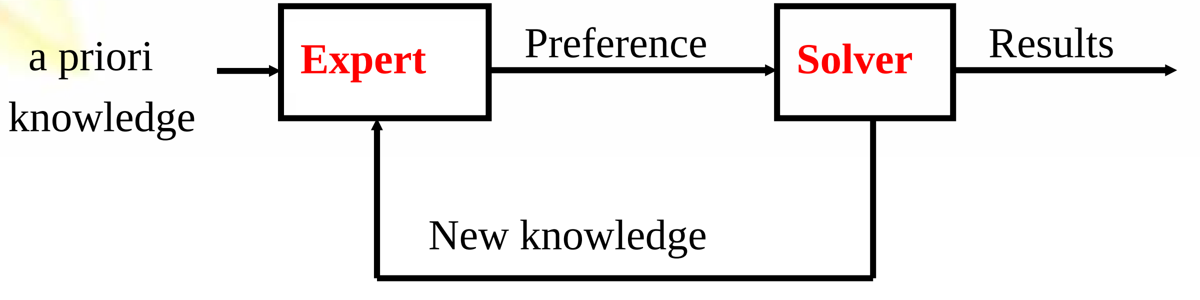
  A priori knowledge of the tackled problem.

- **A posteriori :** **Search** ⟶ **Preference**

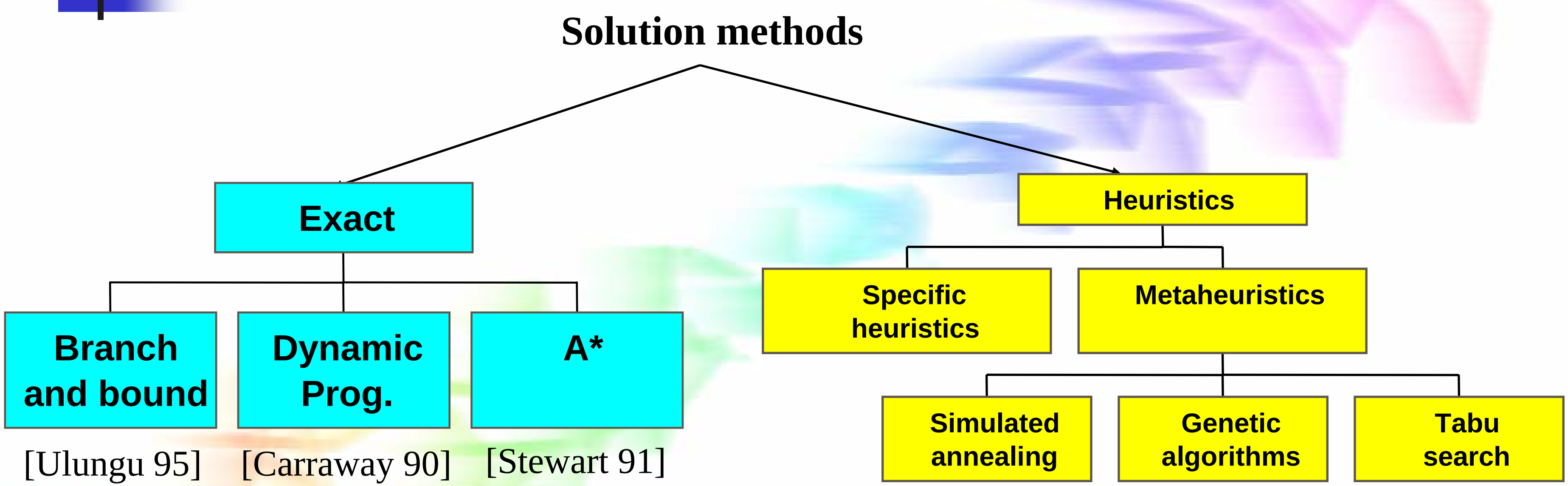
  Cardinality of the reduced PO set.

- **Interactive :** **Search** ⟷ **Preference**

  Decision helping isn't addressed



Taxonomy of optimization methods



Exact algorithms : bi-criterion problems of small size

Metaheuristics :

- solution-based: simulated annealing, tabu search, …
- population-based : genetic algorithms, scatter search, Ant systems, …

PO* : Approximation of PO

Academic MOPs

- Scheduling [Sayin et al. 99]
- Paths [Warburton 87], Spanning tree [Zhou et Gen 99]
- TSP [Serafini 92], Assignment [Teghem 95]
- Vehicle Routing [Park et Koelling 89], …

Multi-criterion knapsack [Teghem et al. 97]

$$\begin{cases} Max\left(f_i(x)\right) = \sum_{j=1}^{m} c_j^i x_j \\ (i = 1,\ldots, n),\ x \in C \\ C = x / \sum_{j=1}^{m} w_j x_j < W \\ x_j \in \{0,1\},\ \forall j = 1,\ldots, m \end{cases}$$

$$x_j = \begin{cases} 1 & \text{if } j \text{ is in the knapsack} \\ 0 & \text{else} \end{cases}$$

w_j : weight of j

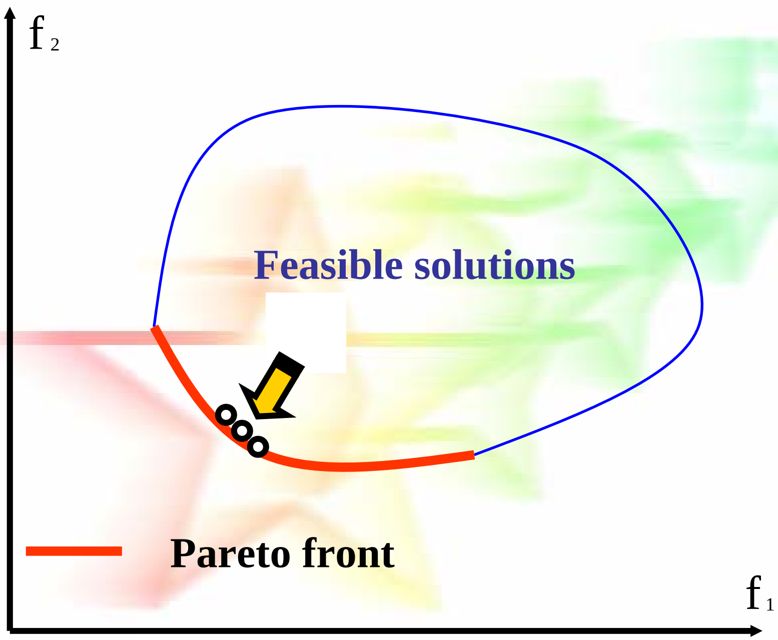
c_j^i : usefulness of j / objective i

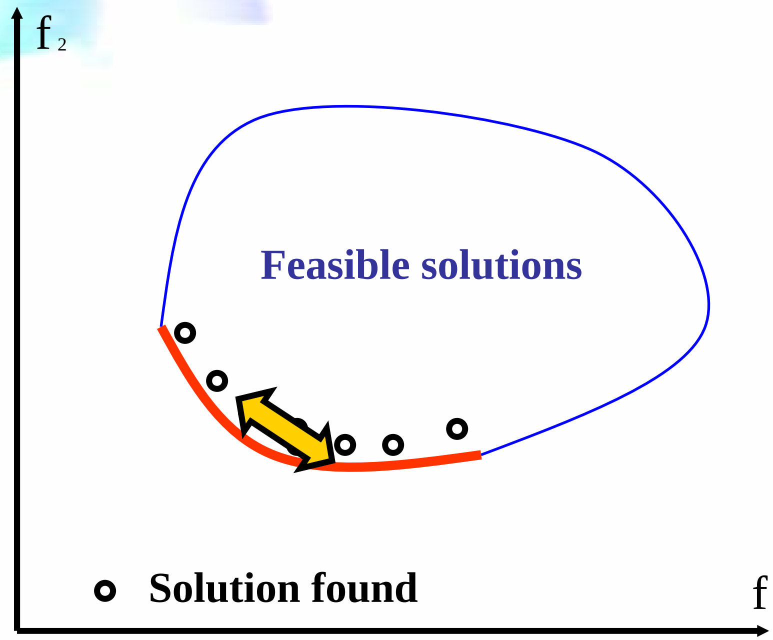
Design of Pareto Search approaches

Goals

Convergence : ranking (order between individuals), elitism,…

Diversification : sharing,…

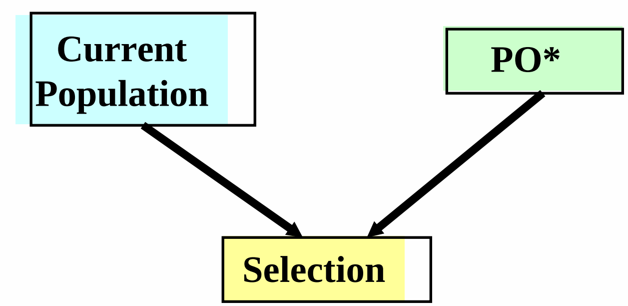




Advanced techniques

- Hybridization
  - Genetic algorithm + Local search (Tabu, Archive Pareto) [Talbi et al. 2002]
- Elitism (archive PO*) [Zitzler et Thiele 98]

$$p_n = \frac{N-A}{N}\,\frac{S(N+1-R_n)+R_n-2}{N(N-1)}$$



A : Number of individuals selected from PO*

- Clustering of the PO* set [Roseman et Gero 85]

E-G. Talbi, M. Rahoual, M-H. Mabed, C. Dhaenens, **« New genetic approach for multicriteria optimization problems : Application to the flow shop »**, *EMO'01, LNCS No.1993, Springer Verlag, pp.416-428, Zurich, Switzerland, Mar 2001.*

Performance evaluation

PO is known

- Absolute efficiency **[Teghem et al.]**

Proportion des solutions Pareto dans PO*

$$AE = \frac{|PO^* \cap PO|}{|PO|}$$

- Distance (PO*, PO)

Plus mauvaise distance

$$WD = \max\left(d(PO^*, y)\right), y \in PO$$

Distance moyenne

$$MD = \frac{\sum_{y \in PO} d(PO^*, y)}{|PO|}$$

$$d(PO^*, y) = \min\left(d(x, y)\right), x \in PO^*$$

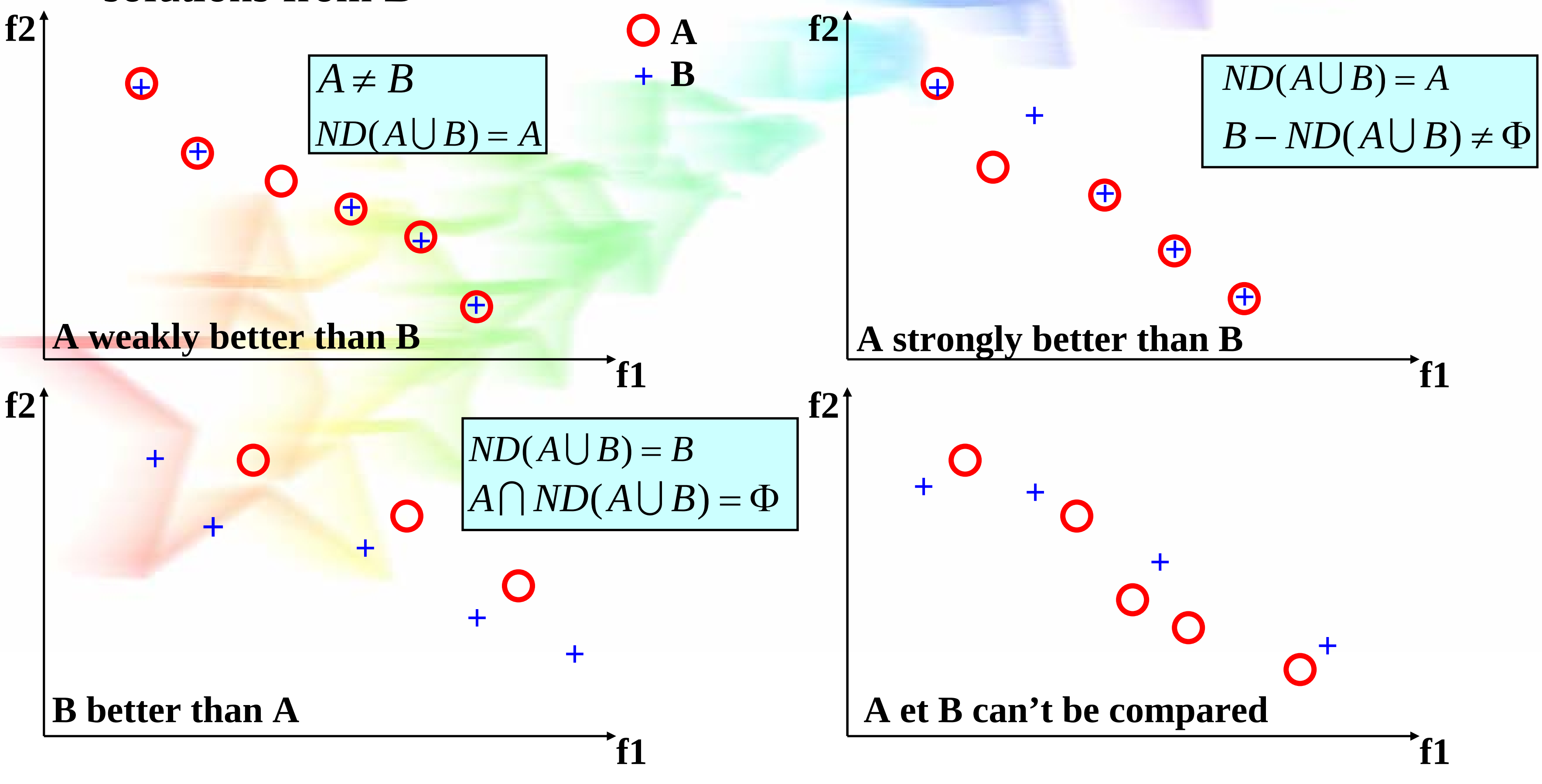
$$d(x,y) = \sum_{i=1}^{n} \lambda_i \left| f_i(x) - f_j(y) \right|$$

- Uniformity

$$DIV = \frac{WD}{MD}$$

PO is unknown

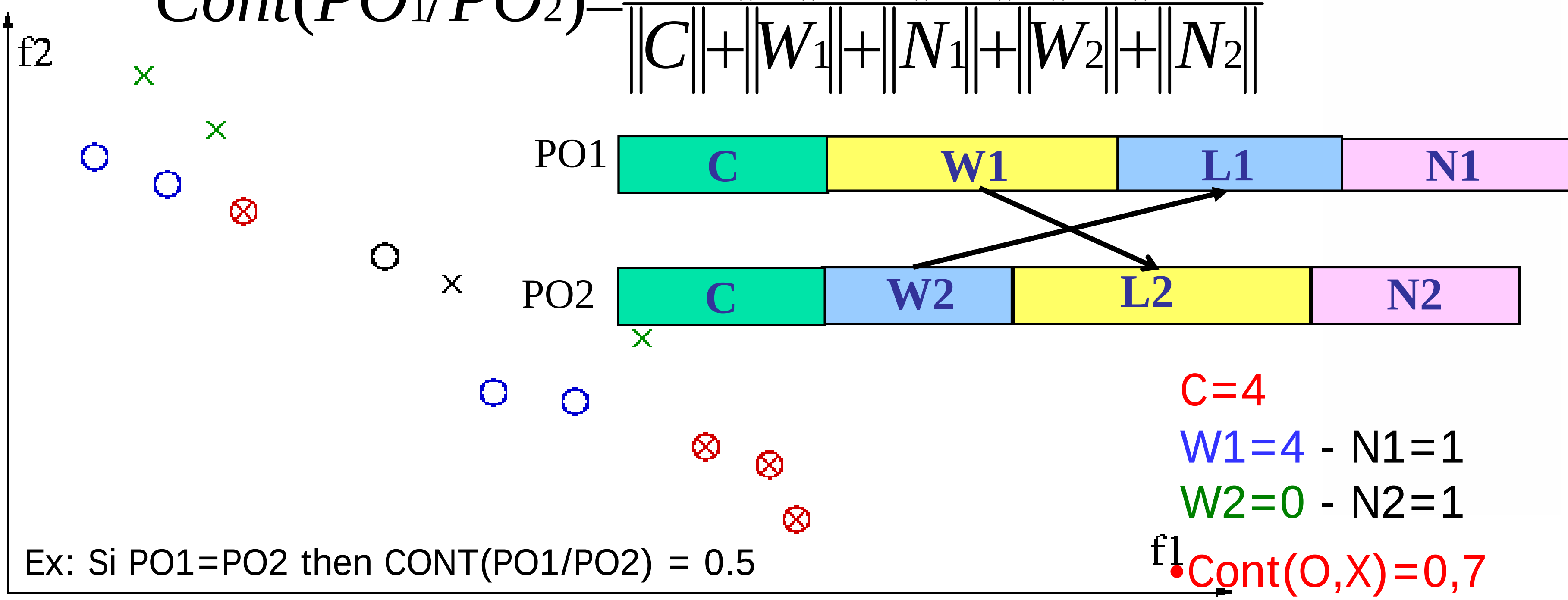
- Relative efficiency (A, B) : Number of solutions from A dominated by solutions from B



PO is unknown

Contribution : Evaluating the quality of the solutions from a set towards another one

$$Cont(PO_1/PO_2)=\frac{\|C\|/2+\|W_1\|+\|N_1\|}{\|C\|+\|W_1\|+\|N_1\|+\|W_2\|+\|N_2\|}$$



Ex: Si PO1=PO2 then CONT(PO1/PO2) = 0.5

Si PO1>PO2 then CONT(PO1/PO2) = 1

C=4

W1=4 - N1=1

W2=0 - N2=1

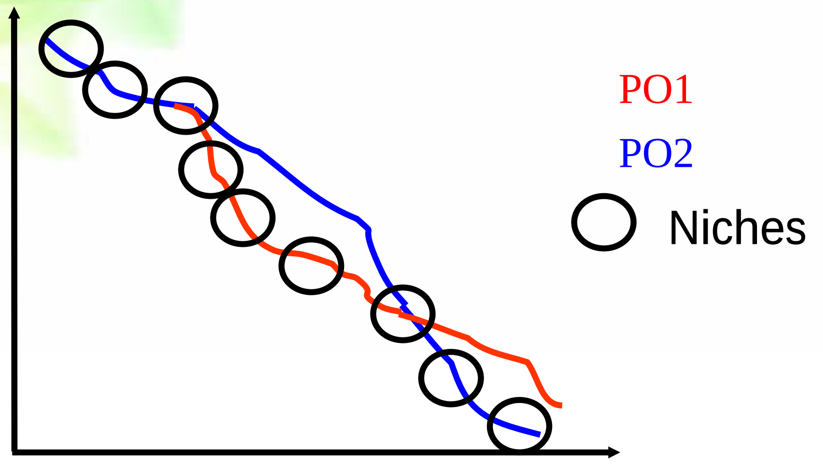
- Cont(O,X)=0,7
- Cont(X,O)=0,3

PO is unknown

Entropy : builds a niche around every solution of ND(PO_1 U PO_2)=PO*

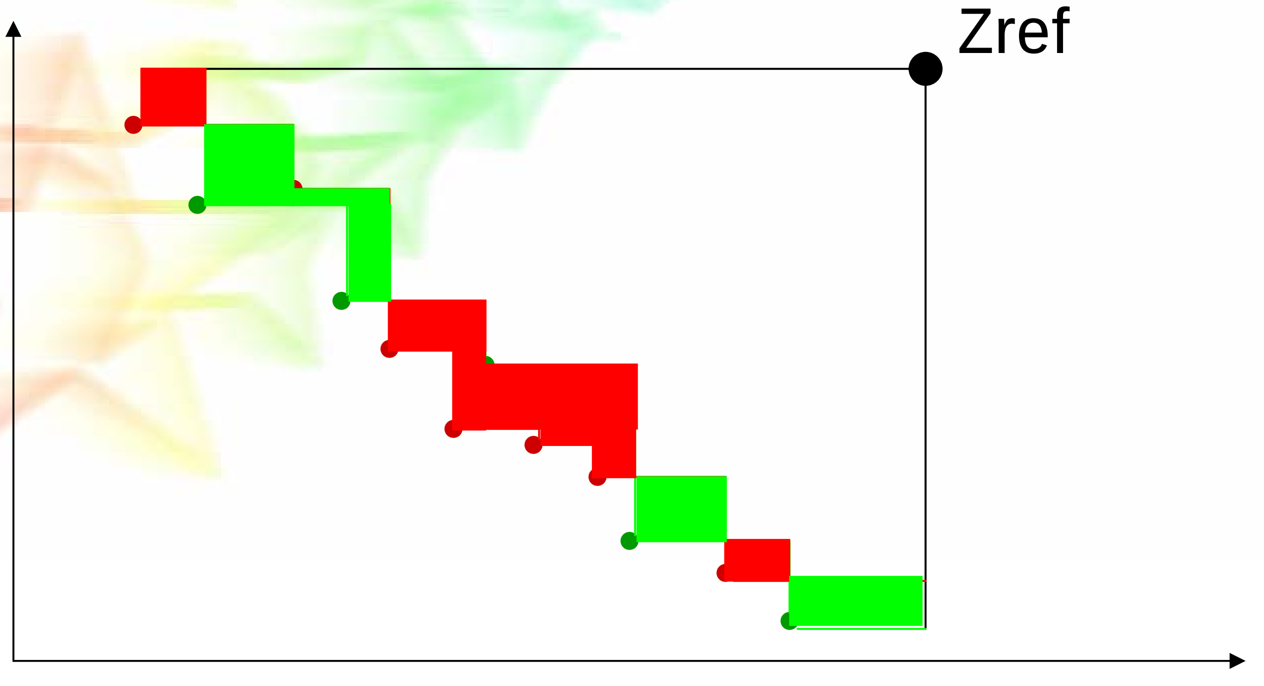
- E(PO_1,PO_2) : diversity of the solutions of PO_1 in comparison of those in the niches of PO*

$$E(PO_1,PO_2)=\frac{-1}{\ln(\gamma)}\sum_{i=1}^{|PO^*|}\left(\frac{1}{Ni}\frac{n_i}{\|PO_1\|}\ln\frac{n_i}{\|PO_1\|}\right)$$



PO is unknown

- S metric *[Zitzler99]:* Compute dominance area enclosed by PO* and a reference point.



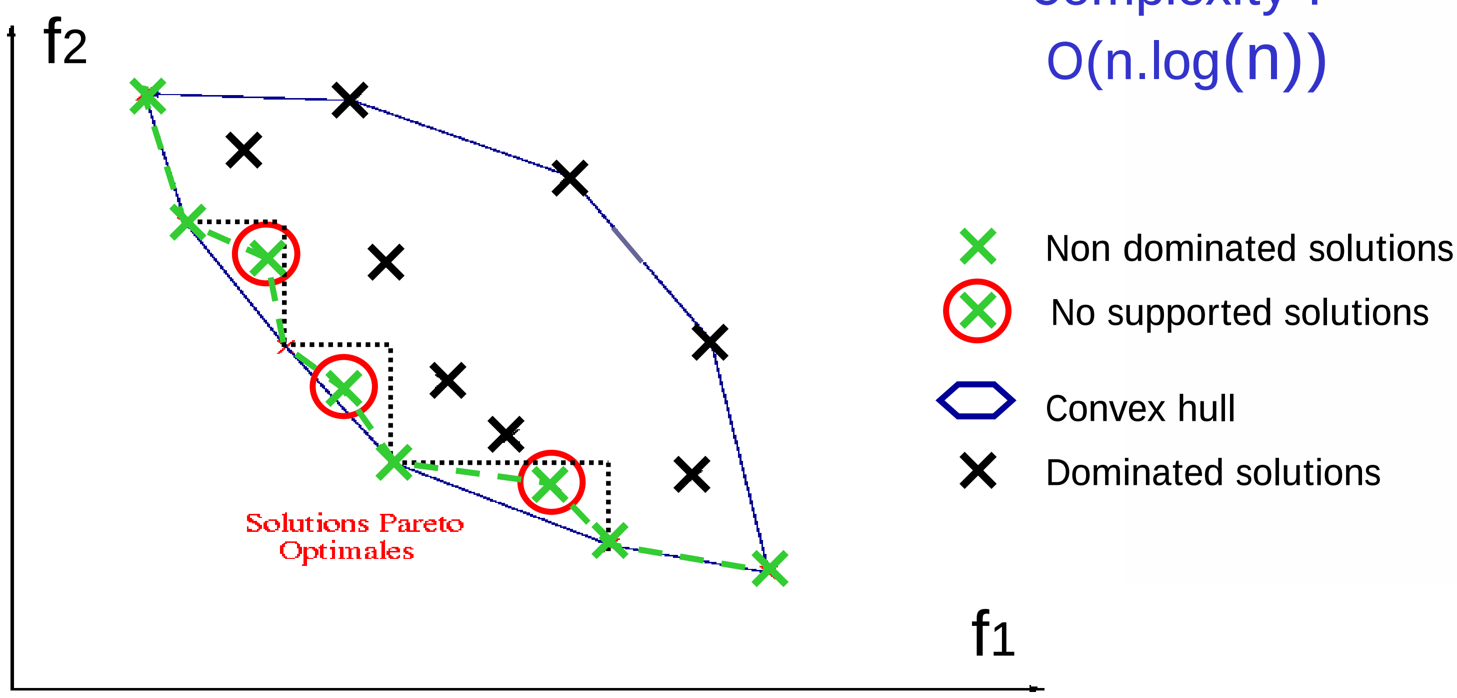
Landscapes

- How to describe a Pareto frontier ?
  - Convexity of PO (supported or not)
  - Multi-modality
  - Deception (deceptive attractors)
  - Isolated optimum (Flat space)
  - Discontinuity
  - Uniform distribution

Landscape

- Aggregation : solutions supported only
- Convexity : Proportion of Pareto solutions belonging to the convex hull

Complexity : O(n.log(n))



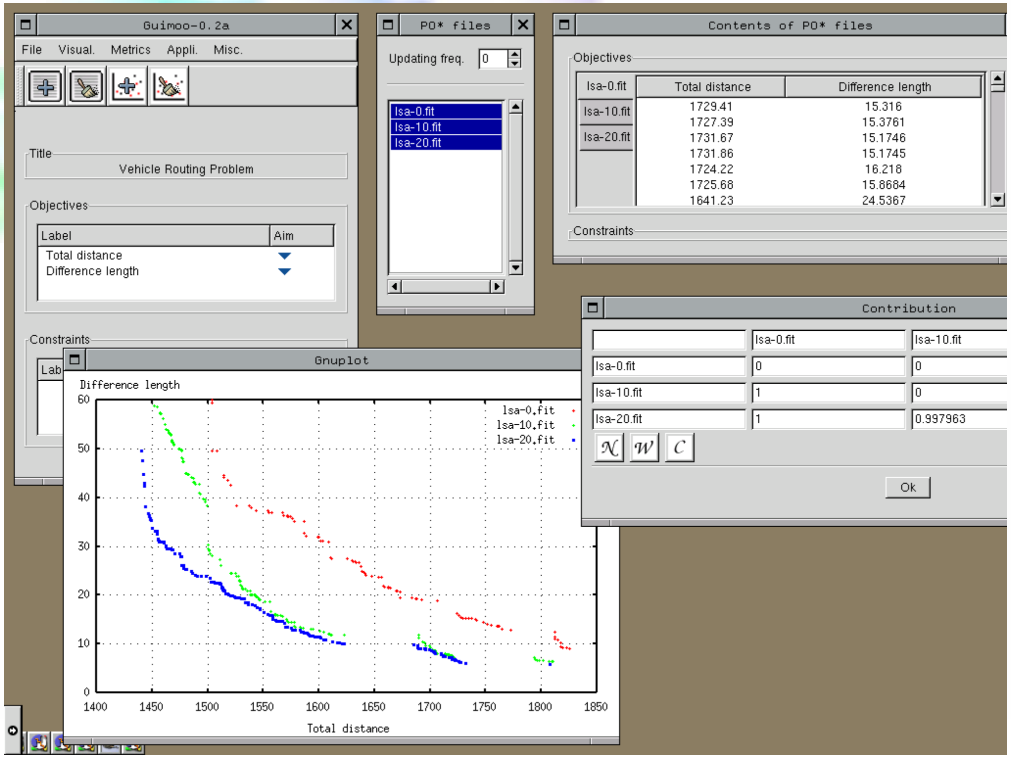
GUIMOO (a Graphical User Interface for Multi-Objective Optimization)

GUIMOO

Free software (open source):

- Performance evaluation of multi-objective optimization methods
- On-line and off-line visualization (2D/3D) of Pareto frontiers

- **Available metrics: Spacing, Coverage, Generational distance, Contribution, Entropy, …**
- **Portability on different systems**
- **Different input/output formats**
- **Easily customized for specific applications (telecom, genomics, engineering design, …)**
- **On-line user manual**



http://www.lifl.fr/OPAC/guimoo

Main features

- Pareto frontiers
  → Display 2/3D (off/on line)
  - Structure
    - Continuous/Discontinuous
    - Concave/Convex
  - Distribution of the solutions in the space of criterion

- Performance analysis
  → Metrics
  - Contribution
  - Entropy (par niches)
  - Generational distance,
  - Spacing,
  - S/D/R metrics