Parallel Cooperative Optimization ***Research Group***

Single Solution-based Metaheuristics

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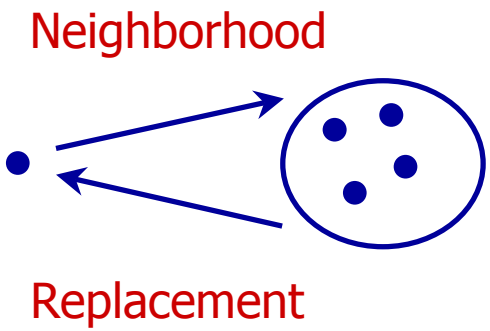
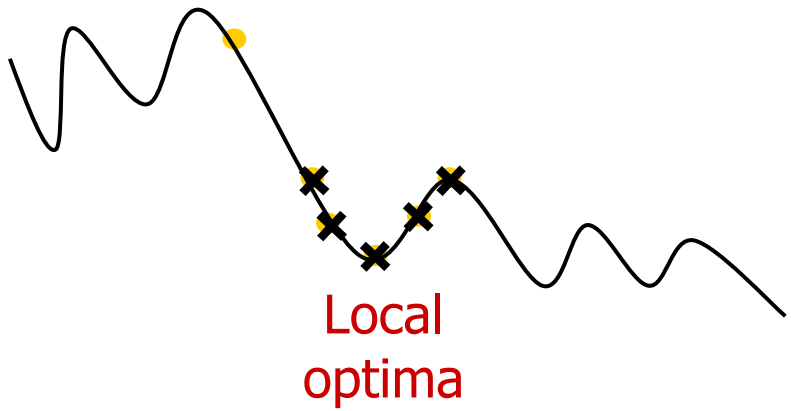
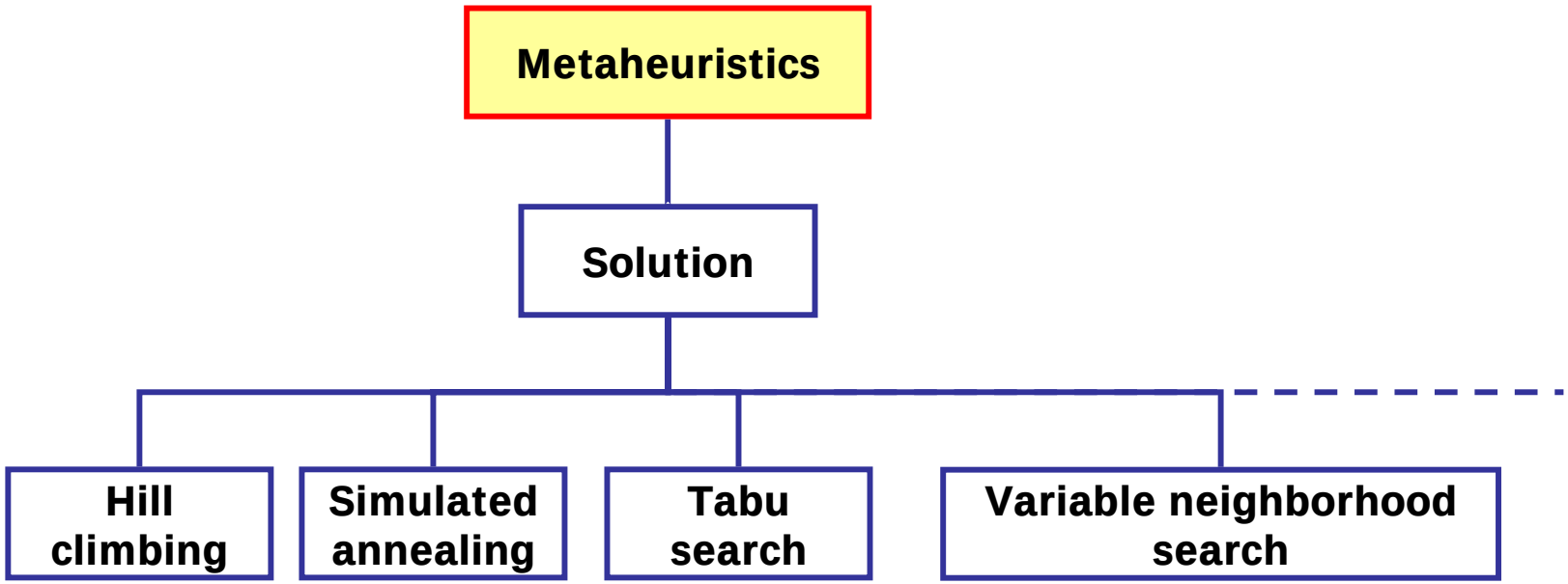
Single solution-based metaheuristics

- “Improvement” of a solution.
- Oriented exploitation:
  - Based on the descent.
  - Exploration of the neighborhood. (intensification)





Taxonomy (Single solution based metaheuristics)



Outline

- Hill Climbing.
- Simulated Annealing.
- Tabu Search.
- Variable Neighborhood search.

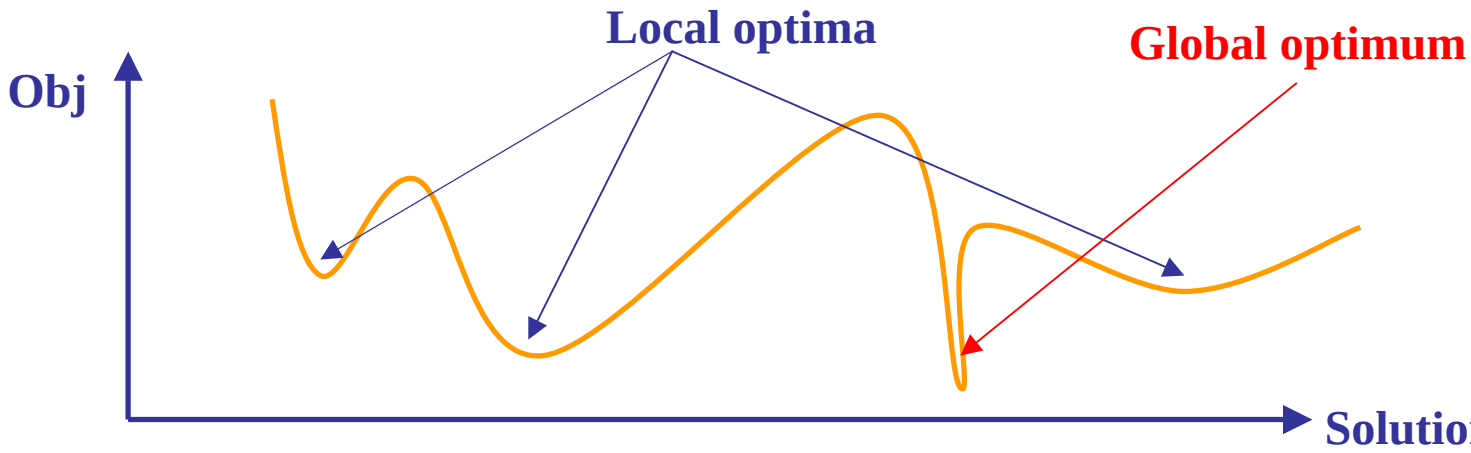
Hill Climbing

Outline

- Hill Climbing:
  - Main characteristics,
  - Algorithm,
  - Trajectory guidance,
  - Efficient evaluation/application of moves.
- Simulated Annealing.
- Tabu Search.
- Variable Neighborhood search.

Hill Climbing

- Easy to implement.
- It only leads to local optima.
- The found optima depends on the initial solution.
- No mean to estimate the relative error from the global optimum.
- No mean to have an upper bound of the computation time.



Strategies for the selection of a neighbour

- Deterministic/full: choosing the best neighbor (i.e. that improves the most the cost function).
- Deterministic/partial: choosing the first processed neighbour that is better than the current solution.
- Stochastic/full: processing the whole neighborhood and applying a random better one.
- ...

Hill Climbing algorithm (1/2)

- **Function** HILL_CLIMBING (*Problem*) **returns** a solution state.
- **Inputs**: *Problem*, a problem.
- **Local Variables**:
  - *Current*, a state,
  - *Next*, a state,
  - *Local,* a boolean.

Hill Climbing algorithm (2/2)

- *Current* = MAKE-NODE(INITIAL STATE[*Problem*]);
- *Local = false;*
- **Do**
  - *Next* = a neighbour of *Current,* selected according to a guidance strategy (det/stoch, full/partial, etc);
  - **If (***Next* is better than *Current***) then**
    - *Current = Next;*
    - Local = false;
  - **Else**
    - Local = true;
- **Until not local**
- **Return** *Current;*

Efficiency issues

- The evaluation function is calculated for every candidate neighbour.
- Often the cost function is the most expensive part of the algorithm.
- Therefore,
  - We need to evaluate the cost function as efficiently as possible:
    - Use Incremental (Delta) Evaluation.

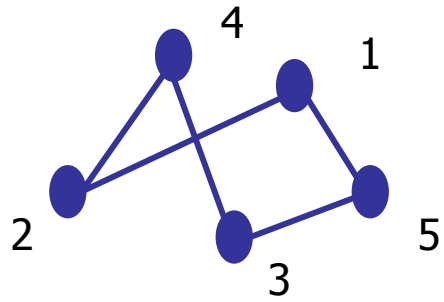
Application to TSP

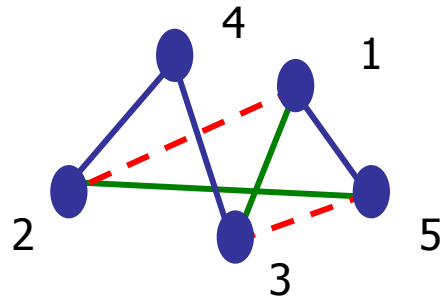
- Example: “Two-opt”:
  - Two points within the permutation are selected and the segment between them is inverted.

➔This operator put two new edges in the tour.

2	1	5	3	4

2	5	1	3	4





Delta = - d(2,1) – d (5,3) + d(2, 5) + d(1, 3)

Hill Climbing Pros & Cons

- Pros:
  - Easy to implement and fast at execution.
- Cons:
  - Get stuck at local optima,
  - Possible solutions:
    - Multi-start: try several runs, starting at different initial solutions,
    - Increasing the size of the neighborhood:
      - In TSP try 3-opt rather than 2-opt.

Simulated Annealing

Outline

- Hill Climbing.
- Simulated Annealing:
  - Main features,
  - Criterion of move acceptation,
  - Cooling schedule.
- Tabu Search.
- Variable Neighborhood search.

Simulated Annealing (1/2)

- Motivated by the physical annealing process.
- Material is heated and slowly cooled into a uniform structure.
- Simulated annealing mimics this process.
- The first SA algorithm was developed in 1953 (Metropolis).
- Kirkpatrick (1982) applied SA to optimization problems:
  - Kirkpatrick, S , Gelatt, C.D., Vecchi, M.P. 1983. "Optimization by Simulated Annealing". Science, vol 220, No. 4598, pp 671-680.

Simulated Annealing (2/2)

- Compared to hill climbing the main difference is that SA allows downwards steps.
- Simulated annealing also differs from hill climbing in that a move is selected at random and its acceptation is conditional:
  - Yet, moves that improve the cost function are always accepted.

The decision criterion in accepting a given move

- The law of thermodynamics states that at temperature, *t*, the probability of an increase in energy of magnitude, δE, is given by:

$$P(\delta E) = \exp(-\delta E / kt)$$

- Where *k* is a constant known as Boltzmann's constant.

Accepting a move or not (1/2)

$P = \exp(-c/t) > r$

- Where:
  - *c* is change in the evaluation function,
  - *t* the current temperature,
  - *r* is a random number between 0 and 1.
- Example:

Change	Temp	exp(-C/T)		Change	Temp	exp(-C/T)
0.2	0.95	0.810157735		0.2	0.1	0.135335283
0.4	0.95	0.656355555		0.4	0.1	0.018315639
0.6	0.95	0.53175153		0.6	0.1	0.002478752
0.8	0.95	0.430802615		0.8	0.1	0.000335463

Accepting a move or not (2/2)

- The probability of accepting a worse state is a function of both the temperature of the system and the change in the cost function.
- As the temperature decreases, the probability of accepting worse moves decreases.
- If t=0, no worse moves are accepted (i.e. hill climbing).

Simulated Annealing algorithm (1/2)

- **Function** SIMUL_ANNEALING(*Problem*) **returns** a solution state
- **Inputs**:
  - *Problem*, a problem,
  - *Schedule*, a mapping from time to temperature.
- **Local Variables**:
  - *Current*, a node,
  - *Next*, a node,
  - *T*, a “temperature” controlling the probability of downward steps,
  - *num_steps*, giving the number of moves performed for a given temperature value.

Simulated Annealing algorithm (2/2)

- $T = T_{init}$;
- *Current* = MAKE-NODE(INITIAL-STATE[*Problem*]);
- **Do**
  - **For** k from 1 to num_steps do
    - *Next* = a randomly selected successor of *Current;*
    - ΛE = VALUE[*Next*] – VALUE[*Current*];
    - **If** ΛE > 0 **then** *Current* = *Next;*
    - **else** *Current* = *Next* only with probability exp(-ΛE/T);
  - **Done**
  - *T* = *Schedule*[*T*];
- **Until** $T < T_{min}$;
- **Return** Current;

The cooling schedule

- The cooling schedule is *hidden* in this algorithm - but it is important (more later) -

- The algorithm assumes that annealing will continue until temperature is zero - this is not necessarily the case:
  - Starting temperature,
  - Final temperature,
  - Temperature update,
  - Iterations at each temperature.

Starting temperature

- *Not* be so hot that we conduct a random search for a period of time.

- *Hot* enough to allow moves to *almost* neighbourhood state (else hill climbing):

  ➔ 60% of worse moves are accepted.

- If we know the maximum change in the cost function we can use this to estimate it.

Choosing the final temperature

- Compromise (Stopping criteria):
  - Low temperature,
  - When the system is “frozen” at the current temperature (i.e. no better or worse moves are being accepted).

Choosing the temperature decrement (1/2)

- Theory ⇔ Number of iterations at each temperature might be exponential to the problem size.

- Compromise:
  - A large number of iterations at a few temperatures,
  - A small number of iterations at many temperatures.

Choosing the temperature decrement (2/2)

- Decrement function in general:

  ➔ *temp* = *temp* * a

  - Experience has shown that a should be between 0.8 and 0.99,

  - Compromise: quality / search time.

Iterations at each temperature

- A constant number of iterations at each temperature (Depend on the Neighborhood size).

- Another method, first suggested by (Lundy, 1986) is to only do one iteration at each temperature, but to decrease the temperature *very* slowly:

$$t = t/(1 + \beta t)$$

  - The formula used is where β is a suitably small value.

- Dynamic:
  - Small number of iterations at high temperature,
  - Large number of iterations at low temperature.

References

- E. Aarts, J. Korst, « Simulated Annealing and Boltzmann Machines », *John Wiley, New York, 1989.*
- E.H.L. Aarts, J.K. Lenstra (eds), « Local Search in Combinatorial Optimization », *John Wiley, Chichester, 1997.*
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Tabu Search

Outline

- Hill Climbing.
- Simulated Annealing.
- Tabu Search:
  - Main features,
  - Management of tabu moves/solutions,
  - Aspiration criterion,
  - Example.
- Variable Neighborhood search.

Main characteristics (1/2)

- Proposed independently by Glover (1986) and Hansen (1986).
- It behaves like Hill Climbing algorithm.
- But it accepts non-improving solutions in order to escape from local optima (where all the neighbouring solutions are non-improving).
- Deterministic algorithm.

Main characteristics (2/2)

- After exploring the neighbouring solutions, we accept the best one even if it decreases the cost function.

  $\rightarrow$ A worse move may be accepted.
- Three goals in the use of memory:
  - preventing the search from revisiting previously visited solutions (tabu list).
  - exploring the unvisited areas of the solution space (diversification).
  - Exploiting the elite (best found) solutions found (intensification).

Features of Tabu Search (1/3)

- Memory related - recency (How recent the solution has been reached):
  - Tabu List (short term memory): to record a limited number of attributes of solutions (moves, selections, assignments, etc) to be discouraged in order to prevent revisiting a visited solution,
  - Tabu tenure (length of tabu list): number of iterations a tabu move is considered to remain tabu.

Features of Tabu Search (2/3)

- Memory related – frequency:
  - Long term memory: to record attributes of elite solutions to be used in:
    - Diversification: Discouraging attributes of elite solutions in selection functions in order to diversify the search to other areas of solution space,
    - Intensification: giving priority to attributes of a set of elite solutions (usually in weighted probability manner).

Features of Tabu Search (3/3)

- If a move is good, but it is tabu, do we still reject it ?
- Aspiration criteria ⇔ accepting a solution even if generated by a tabu move:
  - Best found solution.

Tabu Search algorithm (1/2)

- **Function** TABU_SEARCH(*Problem*) **returns** a solution state.
- **Inputs**: *Problem*, a problem.
- **Local Variables**:
  - *Current*, a state,
  - *Next*, a state,
  - *BestSolutionSeen*, a state,
  - *H*, a history of visited states.

Tabu Search algorithm (2/2)

- *Current* = MAKE-NODE(INITIAL-STATE[*Problem*]);
- **While not terminate**
  - *Next* = a highest-valued successor of *Current;*
  - **If(not** Move_Tabu(*H,Next*) **or** Aspiration(*Next*)**) then**
    - *Current* = *Next;*
    - Update *BestSolutionSeen;*
    - *H* = Recency(*H* + *Current);*
  - Endif
- **End-While**
- **Return** *BestSolutionSeen;*

Example: TSP using Tabu Search (1/3)

	V_1	V_2	V_3	V_4
V_1	X			
V_2		X		
V_3			X	
V_4				X

- In our example of TSP:
  - Short term memory:
    - Maintain a list of *t* edges and prevent them from being selected for consideration of moves for a number of iterations.
    - After a number of iterations, release those edges.
    - Make use of a squared table to decide if an edge is tabu or not.

Example: TSP using Tabu Search (2/3)

- In our example of TSP:
  - Long term memory:
    - Maintaining a list of *t* edges which have been considered in the last *k* best (worst) solutions
    - encouraging (or discouraging) their selections in future solutions.
    - using their frequency of appearance in the set of elite solutions and the quality of solutions which they have appeared in our selection function.

Example: TSP using Tabu Search (3/3)

- In our example of TSP:
  - Aspiration:
    - If the next moves consider those moves in the tabu list but generate better solution than the current one,
    - Accept that solution anyway,
    - Put it into tabu list.

Tabu Search Pros & Cons

- Pros:
  - Generate generally good solutions for optimisation problems compared with other solution based metaheuristics.
- Cons:
  - Tabu list construction is problem specific,
  - Long term memory is problem specific,
  - Different parameters (size tabu list, …).

References

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- Glover, F., Laguna, M. 1998. *Tabu Search*. Kluwer Academic Publishers.
- Rayward-Smith, V.J., Osman, I.H., Reeves, C.R., Smith, G.D. 1996*. Modern Heuristic Search Methods*, John Wiley & Sons.
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- R. Qu, "Artificial Intelligence methods: Tabu search".

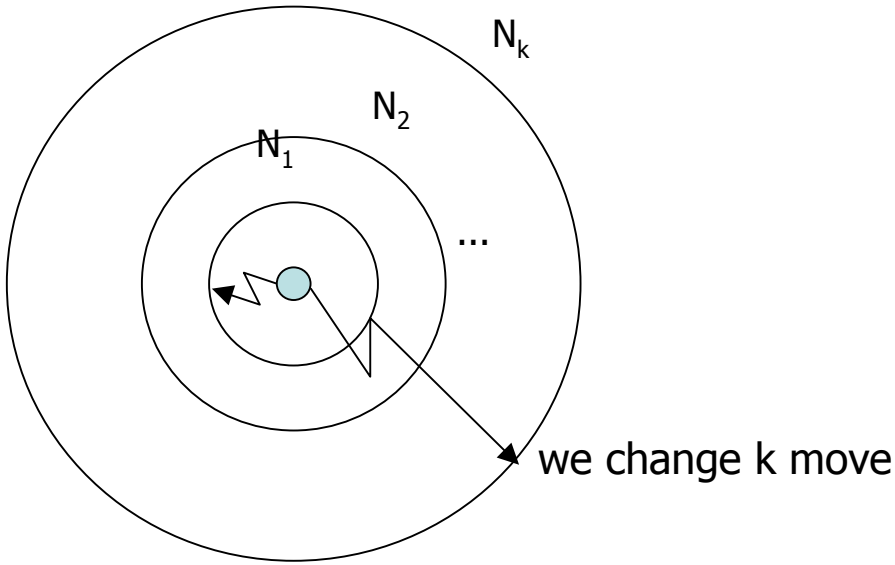
Variable Neighborhood-search

Variable Neighborhood-search (1/3)

- Inspired from Hill Climbing.
- But manages several neighborhoods (hence different move concepts).

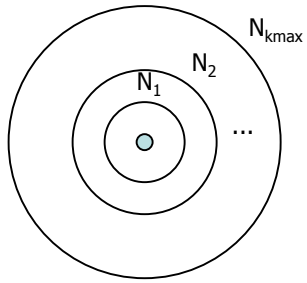
Variable Neighborhood-search (2/3)

- Definitions:
  - Nk → Neighborhood in k distance:
    - Distance: how many locations we change.
  - Nk(x) → Set of solutions in the kth neighbourhood of x.



Variable Neighborhood-search (3/3)

- **Initialisation:**
  - Select the set of neighbourhood structures Nk,
  - Find an initial solution x.



- **Repeat** until stopping condition is met
  - Set K=1;
  - **Repeat** until k=kmax
    - *Shaking*: Generate a random point X' in Nk(x);
    - *Local Search*: x'' is the obtained optimum;
    - Move or not:
      - If x'' is better than x then x=x'' and k=1;
      - Otherwise k=k+1;